De Re Mathematical Necessities: An Aristotelian Explanation

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Part One: Mathematically Necessary Facts About Reality

Examples:

- 1. It is impossible for someone to walk over all the Konigsberg bridges crossing each exactly once. (Pincock 2007)
    - Because there is no such path through a K-graph.
- 2. This room's floor cannot be tiled with regular, equal pentagons. (Franklin 2014)
    - Because the Euclidean plane cannot be tiled by regular, equal pentagons.
- 3. Necessarily, any body that is symmetric about both axes is symmetrical about the point of intersection of the axes. (Franklin 2014)
- 4. Five apples cannot be equally divided among three people. (Braine 1972)
    - Because the number 5 is not divisible by the number 3.
- 5. This trefoil knot cannot be unknotted without cutting. (Lange 2017)
    - Because a trefoil knot in 3D space is not isotopic to the unknot.

Strong Mathematical Modality:

- In these cases, the necessity that applies to the relevant facts seems to be a very *strong* type of modality: *mathematical modality*
    - To tile a floor with pentagons, or to walk each bridge of Konigsberg exactly once, do not seem to just be very difficult things to do. These seem, in some sense, positively *incoherent*.
        - (But these are not, in fact, strictly *logical* contradictions!)
    - Doing these things seems more impossible than even a violation of some physical law – e.g., a violation of the conservation of energy.
        - *Even if* conservation were violated, my necessities would hold.
    - Doing these things seems more impossible even than violations of some *metaphysical* laws: e.g., a contingent being, like a pink elephant, popping into being out of nothing, with no cause.

Part Two: 'Platonic' Explanations of De Re Mathematical Necessity

- "Platonism" (so-called): Mathematical truths are about a realm of necessary, spaceless, timeless, causally inefficacious abstract objects.
- **The Gap Problem**: Even if we grant that facts about the Platonic horde are metaphysically necessary (perhaps in virtue of the natures of the relevant Platonica), that doesn't explain the necessity of the relevant *physical* facts, which have to do with totally *separate* objects.

Part Three: A Neo-Aristotelian Explanation

- Immanent Structuralism: Mathematics does not study some special class of abstract mathematical objects. Rather, mathematics studies a particular class of *universals* or *properties* – the purely structural properties – and these can be *literally instantiated* by physical things.
- **Definition**: P is a *purely structural property* iff P can be defined entirely in terms of 'part', 'whole', 'sameness', 'difference', and logical vocabulary.
- **Example**: The property of being a *K-graph* [recall the Konigsberg bridges] is the property of being a whole G with some distinct parts $v_1, \ldots, v_4$, and some relation E between these parts such that v_1Ev_2, v_1Ev_3, … (etc.).
- Constitutive parthood statements:
  - **being a mammal $\Subset$ being a dog**
- A Property-Parthood Account of Mathematical Truth: "0 has a successor"
  - **having a successor $\Subset$ being the object that is not a successor**
  - **"The thing that is not a successor ('the number 0') has a successor."**
- I view property parthood statements as equivalent to *essence* statements:
  - **Being a mammal** is essential to (is "part" of the essence of) **Being a dog**. Or, to use my symbolism: **Being a mammal $\Subset$ Being a dog**
- One can thereby explain why any mammal (e.g. Paco) <u>**must**</u> be an animal: Because ***being an animal*** is part of the essence of ***being a mammal.***
- Similarly, I can explain why the Konigsberg bridges <u>**cannot**</u> be crossed once each: Because the Konigsberg bridges instantiate being a K-graph, and ***having no Eulerian path*** is part of the *essence* of ***being a K-Graph***.